Article

Determining an Accurate and Cost-Effective Individual Height-Diameter Model for Mongolian Pine on Sandy Land

Yangang Han [1,2,3,4], Zeyong Lei [1,3,*], Albert Ciceu [5,6], Yanping Zhou [7], Fengyan Zhou [1,3] and Dapao Yu [2,*]

1   College of Environmental Science and Engineering, Liaoning Technical University, Fuxin 123000, China; hyghan@163.com (Y.H.); fengyanzhou@sohu.com (F.Z.)
2   CAS Key Laboratory of Forest Ecology and Management, Institute of Applied Ecology, Shenyang 110016, China
3   Institute of Land Degradation and Ecological Restoration, Liaoning Technical University, Fuxin 123000, China
4   University of Chinese Academy of Sciences, Beijing 100049, China
5   Department of Forest Monitoring, "Marin Drăcea" Romanian National Institute for Research and Development in Forestry, 128 Eroilor Blvd., 077190 Voluntari, Romania; albert.ciceu@icas.ro
6   Department of Forest Engineering, Forest Management Planning and Terrestrial Measurements, Faculty of Silviculture and Forest Engineering, "Transilvania" University, 1 Ludwig van Beethoven Str., 500123 Braşov, Romania
7   Fuxin Water Affairs Service Center, Fuxin 123000, China; zyping5460@163.com
*   Correspondence: leizeyong@lntu.edu.cn (Z.L.); yudp2003@iae.ac.cn (D.Y.)

Abstract: Height-diameter (H-D) models are important tools for forest management practice. Sandy Mongolian pine plantations (*Pinus sylvestris* var. *mongolica*) are a major component of the Three-North Afforestation Shelterbelt in Northern China. However, few H-D models are available for Mongolian pine plantations. In this paper we compared different equations found in the literature for predicting tree height, using diameter at breast height and additional stand-level predictor variables. We tested if the additional stand-level predictor variable is necessary to produce more accurate results. The dominant height was used as a stand-level predictor variable to describe the variation of the H-D relationship among plots. We found that the basic mixed-effects H-D model provided a similar predictive accuracy as the generalized mixed-effects H-D model. Moreover, it had the advantage of reducing the sampling effort. The basic mixed-effects H-D model calibration, in which the heights of the two thickest trees in the plot were included to calibrate the random effects, resulted in accurate and reliable individual tree height estimations. Thus, the basic mixed-effects H-D model with the above-described calibration design can be an accurate and cost-effective solution for estimating the heights of Mongolian pine trees in northern China.

Keywords: height-diameter; mixed-effects model; dominant height; model calibration; *Pinus sylvestris* var. *mongolica*

1. Introduction

Shelterbelt establishments are an effective solution in preventing natural disturbances and hazards such as soil erosion, avalanches or landslides [1]. The Three-North Shelter Forest Program is the largest artificial forest shelter program in the world [2]. This shelter forest program plays a vital role in carbon sequestration and sand fixation, soil conservation, wood production and supply for the local communities [3]. By 2018, a forest area plantation of 3.01×10^7 hm^2 had remained from the Three-North Shelter Forest Program [3]. Sandy Mongolian pine (*Pinus sylvestris* var. *mongolica*) is one of the most important and common species used for afforestation in the Three-North Shelter Forest Program. The area of sandy Mongolian pine plantations in northern China exceeds 7×10^5 hm^2 [4].

Tree height and diameter at breast height are two elementary input variables used in forest management practice [5,6], with tree height being an input variable needed in

most allometric equations that estimate tree biomass and volume [7–9]. Furthermore, tree height estimates are needed in growth and yield modelling. However, tree height measurements are difficult, time-consuming and laborious, while tree diameters at breast height are relatively easy to measure. Therefore, heights are generally estimated by H-D models based on a subsample of height measurements [10].

Until now, many H-D models with various mathematical forms have been described for different tree species in various forest types around the world [5,8,11–14]. Although the results of these studies have indicated differences in H-D models between species and forest types, general sigmoidal or nonlinear growth curves are suitable for H-D model development [15–17]. Furthermore, the H-D relationship varies between stands of the same species due to stand site quality, stand age, and competition [16,18,19]. In order to reduce the H-D relationship variation between stands, various stand-level predictor variables are usually introduced into a simple basic H-D model, with the aim of developing a generalized H-D models [20,21]. Usually, generalized H-D models are developed at a regional scale, providing the necessary prediction accuracy at the stand level by reducing at the same time the sampling effort needed to build simple H-D models for each stand [17,22,23].

The forestry data are hierarchical, and the traditional ordinary least squares (OLS) regression models always induce biased results in parameter estimation and height predictions. Nonlinear mixed-effects models have, therefore, become increasingly popular in the development of regional H-D models [5,24]. Many studies have reported better goodness-of-fit values and higher prediction accuracies for nonlinear mixed-effects H-D models compared to OLS regression models [25,26].

Mixed-effects models can easily account for the variations in H-D relationships among stands by including stand-level random effect parameters [27,28]. Furthermore, less sampling effort is required and similar accuracy is achieved by calibrating H-D mixed-effects models compared with the sampling effort necessary to build a local H-D model [6,15]. However, stand-level predictor variables may not always be necessary in mixed-effects H-D models' modelling framework. Previous research has demonstrated that the inclusion of additional stand-level predictor variables did not improve the model's predictive ability and accuracy under the mixed-effects framework [5,25]. Hence, a high accuracy can also be obtained by using a parsimonious model that requires less sampling effort. Considering these aspects, a practical approach is needed to establish if the inclusion of additional predictors is really required to obtain the desired accuracy.

Considering the important ecological and economic functions of Mongolian pine plantations in North China, an efficient management system is required based on appropriate scientific tools. To the best of our knowledge, an appropriate H-D model and information on the best calibration design for this species have not yet been well described in the scientific literature. Thus, H-D models are needed for this species not only for volume and biomass estimations but also for future yield and growth models. Here, we hypothesized that: (1) generalized mixed-effects models have better performance than basic mixed-effects models both in fitting and calibration; (2) the model calibration performance will improve with the number of prior tree height measurement increasing; and (3) tree height measured from different diameter classes would help improve the model calibration performance.

2. Material and Methods

2.1. Study Area

The data used in this study were collected from plantations in Northwest Liaoning Province, northeast China (42°39′~42°43′ N, 122°22′~122°33′ E), which is part of the Three-North Shelter Forest Program. Autumns and winters in this region are dry, the multiyear mean annual temperature is approximately 5.3 °C, and the multiyear annual precipitation ranges from 300 to 500 mm [29]. The major local vegetation consists of *P. tabuliformis*, *Agriopyllum squarrosum*, *Salix gordejevii*, *Artemisia halodendron*, and *Calamagrostis epigeios* [30].

2.2. Ssampling Design and Data Collection

For this research thirty-five temporary sample plots of 400 m^2 (20 m × 20 m) were established in Mongolian pure pine stands. The sampling plots cover different stand ages and stand structures of the Mongolian pine plantations. The stand age ranges from 13 to 62 years, and includes young, half-mature, mature, and old forests. For each stand age the sampling plots include different stand densities. The detailed sampling workflow can be found in [29,30]. Stand age was obtained for each plot based on the afforestation records. Stand density was registered as the counts of living trees. For all living and healthy trees in each plot, the diameters at breast height (D: 1.3 m above ground) were measured using diameter tape with a precision of 0.1 cm; tree height (H) of all living trees was measured using an infrared dendrometer (Criterion™ RD1000) with a precision of 0.1 m.

Only the trees with D ≥ 5 cm were used in this analysis. A total of 1070 paired height-diameter measurements were chosen for the model development. This dataset was used for the parameter estimation and model calibration. The distributions of the height and diameter at breast measurements are illustrated in Figure 1.
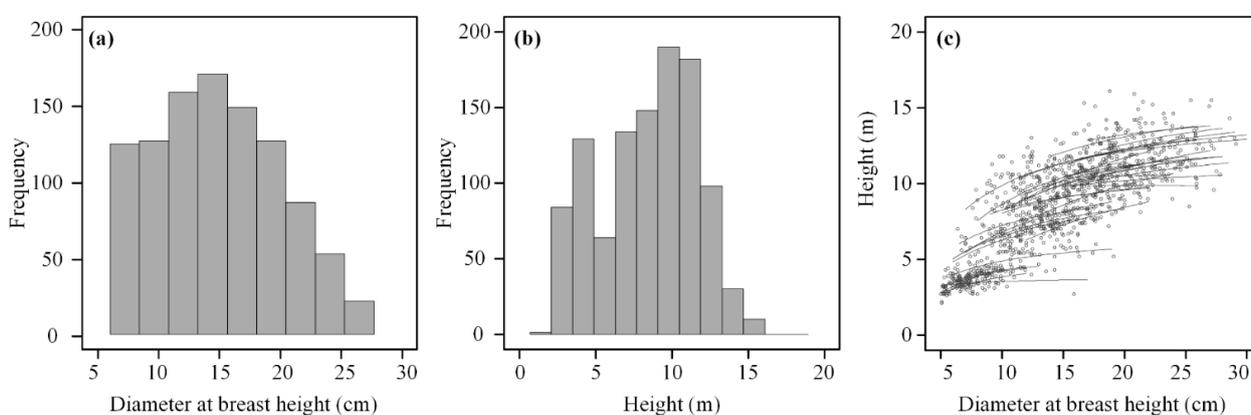
Figure 1. Distributions of the diameter at breast height (**a**) and height (**b**) as well as plot-specific fitted height-diameter lines (**c**). Plot-specific line was obtained by fitting height-diameter (H-D) relationships in each plot using the best basic model determined below.

2.3. Basic Model Selection

As reported in previous studies, nonlinear curves are more suitable than linear ones for representing H-D relationships [15,16]. Following these findings and examining the distributions of our H-D dataset, we chose 8 nonlinear candidate H-D models (Table 1). To ensure parsimony and convergence for each plot, only models with two parameters were selected. The suitability of all candidate models in predicting height was validated in previous research papers [12,31]. The models' performances were compared using the coefficient of determination (R^2), the mean absolute error (MAE), and the root mean squared error (RMSE). The expressions of R^2, MAE, and RMSE are shown in Equations (9)–(11):

Table 1. Candidate height-diameter models.

Model	Formula	References	Equation No
M1	$H = 1.3 + a[D/(1 + D)]^b + \varepsilon$	Curtis [8]	(1)
M2	$H = 1.3 + aD + bD^2 + \varepsilon$	Curtis [8]	(2)
M3	$H = 1.3 + aD^b + \varepsilon$	Stoffels and Soest [32]	(3)
M4	$H = 1.3 + a \exp(b/D) + \varepsilon$	Schumacher [33]	(4)
M5	$H = 1.3 + \exp[a + b/(D + 1)] + \varepsilon$	Wykoff, et al. [34]	(5)
M6	$H = 1.3 + aD/(b + D) + \varepsilon$	Bates and Watts [35]	(6)
M7	$H = 1.3 + 10^a D^b + \varepsilon$	Larson [36]	(7)
M8	$H = 1.3 + a[1 - \exp(-bD)] + \varepsilon$	Meyer [37]	(8)

H is the height, D is the diameter at breast height, a and b are the model coefficients, ε is the error term.

$$R^2 = 1 - \sum_{i=1}^{m} \sum_{j=1}^{n_i} \left(H_{ij} - \hat{H}_{ij} \right)^2 \Bigg/ \sum_{i=1}^{m} \sum_{j=1}^{n_i} \left(H_{ij} - \overline{H} \right)^2 \tag{9}$$

$$\text{MAE} = \frac{1}{n} \sum_{1}^{m} \sum_{1}^{n_i} |H_{ij} - \hat{H}_{ij}| \tag{10}$$

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{1}^{m} \sum_{1}^{n_i} \left(H_{ij} - \hat{H}_{ij} \right)^2} \tag{11}$$

where R^2 is the coefficient of determination; MAE is the mean absolute error; RMSE is the root mean squared error; H_{ij} and $\hat{H}_{ij}$ are the observed and predicted heights of tree j on plot i, respectively; $\overline{H}$ is the average of the observed heights; n_i is the number of trees in the plot i; m is the total number of sample plots; n is the total number of trees.

2.4. H-D Generalized Model

Additional stand-level predictor variables were added to the best basic H-D model in order to develop the generalized H-D model. Among the stand-level predictor variables considered were stand age (Age), stand dominant height (DH), stand basal area (BA), stand density (N), relative spacing (RS), stand quadratic mean diameter (Dg), stand arithmetic mean diameter (AMD), maximum diameter (Dmax), minimum diameter (Dmin), diameter range per plot (Dr) (Table 2). In the present work, DH was defined as the mean height of the 100 largest-diameter trees per hectare [10], as there are no available dominant height models developed for this species.

Table 2. Summary statistics of evaluated stand-level predictor variables.

Variables	Minimum	Mean $\pm$ SD	Maximum
Age (year)	13	36 $\pm$ 14	62
DH (m)	3.2	9.7 $\pm$ 3.0	13.6
BA (m^2 ha^{-1})	2.92	14.70 $\pm$ 6.12	33.34
N (trees ha^{-1})	300	774 $\pm$ 461	2500
RS	1.03	0.45 $\pm$ 0.18	0.16
Dg (cm)	7.7	16.7 $\pm$ 5.0	23.9
AMD (cm)	6.7	16.4 $\pm$ 5.1	23.7
Dmax (cm)	10.0	22.0 $\pm$ 5.4	29.1
Dmin (cm)	1.9	11.2 $\pm$ 4.8	18.0
Dr (cm)	5.7	10.9 $\pm$ 2.2	16.5

SD means standard deviation. Age is the stand age; DH is the dominant height; BA is the basal area per hectare; N is the stand density; RS is the relative spacing; Dg is the stand quadratic mean diameter; AMD is the stand arithmetic mean diameter; Dmax and Dmin are the maximum and minimum diameter of the plot, respectively; Dr is diameter range per plot.

To obtain a generalized model with a high accuracy and an easier biological interpretation, a two-stage reparameterization method was used [38]. In the first step linear regression was conducted using the estimated coefficients for each plot as response variables and the stand-level predictor variables as independent variables. The variables with variance inflation factor (VIF) > 5 were removed to eliminate multicollinearity. Finally, the remaining and most significant variables were used to develop the generalized H-D model.

2.5. Nonlinear Mixed-Effects Model

Based on the OLS basic and generalized models we developed nonlinear mixed-effects models taking into account the hierarchical structure of the data. Additional random parameters associated with the equation's coefficients were considered.

The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) were used to determine the prediction gain by including additional random effects in the following general form of the mixed-effects model [28]:

$$y_i = f(\beta, u_i, x_i) + \varepsilon_i \tag{12}$$

where x_i and y_i are the response and predictor variables, respectively; f is a nonlinear function between the response and predictor variables; β is a $q \times 1$ fixed parameter vector; u_i is a random effects parameter for plot i and is assumed to be distributed normally with variance-covariance matrix Ψ, i.e., $u_i \sim N(0, \Psi)$; ε_i is error term and is assumed to be normally distributed with the within-sample-plot variance-covariance matrix for R_i, i.e., $\varepsilon_i \sim N(0, R_i)$.

The expression of R_i is as follows [39]:

$$R_i = \sigma^2 G_i^{0.5} \Gamma_i \, G_i^{0.5} \tag{13}$$

where σ^2 is the scale; G_i is a diagonal matrix describing the heteroskedasticity; and Γ_i is an autocorrelation structure of error and is an identity matrix in this study.

Three variance functions: exponential, power, and constant plus power, with D as the predictor, were compared in accounting for the heteroscedasticity. These formulas are shown below:

$$var(\varepsilon_i) = \sigma^2 \exp(2\delta D_{ij}) \tag{14}$$

$$var(\varepsilon_i) = \sigma^2 D_{ij}^{2\delta} \tag{15}$$

$$var(\varepsilon_i) = \left(\delta_1 + D_{ij}^{\delta_2}\right)^2 \tag{16}$$

where σ^2 is the scale, and δ is the parameter to be estimated.

2.6. Parameter Estimation

The parameters of the basic and generalized H-D models were first estimated with OLS method using the *nls* function in R software [40]. Parameters of the mixed-effects models were estimated in the mixed-effects modelling framework using the *nlme* function and the restricted maximum likelihood (REML) method in the nlme package in R software [40,41].

2.7. Evaluation of Model Prediction Performance and Appropriate Calibration Design

The mixed-effects models were used for the following types of height predictions in the stand of the entire data:

(1) Fixed-effects prediction of the mixed-effects model, where only the fixed part was used as follows:

$$H_i = f(\hat{\beta}, 0, x_i) + \varepsilon_i \tag{17}$$

where H_i and x_i are the height and predictor vectors of plot i; $\hat{\beta}$ is the vector of estimated fixed parameters; ε_i is error term.

(2) Random-effects prediction of the mixed-effects model for a specific plot.

The random-effects predictions require the random-effects parameters u_i to be calculated with the following Equation [42]:

$$\hat{u}_i = \hat{\psi} \hat{Z}_i^T (\hat{Z}_i \hat{\psi} \hat{Z}_i^T + \hat{R}_i)^{-1} \left[H_i - f(\hat{\beta}, 0, x_i) \right] \tag{18}$$

where $\hat{u}_i$ is the estimated random effects parameters; $\hat{\psi}$ and $\hat{R}_i$ are the estimations of Ψ and R_i, respectively; $\hat{Z}i$ is the design matrix for the random effects; and $\hat{Z}_i^T$ is the transpose matrix of $\hat{Z}_i$.

In this paper, $\hat{u}_i$ was calculated using the empirical best linear unbiased prediction (EBLUP) method [43].

The calibration design involved different sampling strategies. The tree size categories sampled include the thinnest, medium-sized and thickest trees. A total of 4 designs with 47 combinations were evaluated in the random-effects predictions.

(i)   One-six (in order) trees from the same diameter size category were selected.
(ii)   One-three (in order) trees were chosen separately from two diameter size categories.
(iii) One tree from each of the three diameter size categories was selected separately.
(iv)   Two trees were selected separately from each of the three diameter size categories.

In order to determine the reliable and most accurate calibration we first used the paired Wilcoxon rank-sum test to test which calibration design leads to an unsignificant difference between predicted and observed heights. Then, the best calibrations obtained for each number of trees sampled were further compared using MAE and RMSE to determine the best calibration solution for the two mixed-effects models developed.

3. Result

3.1. Basic and Generalized Fixed Model

Although the performances did not drastically differ among the eight models (Table 3), model 4 (M4) had the highest R^2, and the lowest MAE and RMSE values. In addition, M4 could also fit each plot in our dataset (Figure 1c). Thus, M4 was selected as the basic model for further model development.

Table 3. The coefficient of determination (R^2), the mean absolute error (MAE), and the root mean squared error (RMSE) for basic height-diameter model.

Model	R^2	MAE	RMSE
M1	0.6927	1.3573	1.7381
M2	0.6654	1.4738	1.8136
M3	0.6595	1.4848	1.8296
M4	0.6936	1.3526	1.7357
M5	0.6916	1.3631	1.7412
M6	0.6650	1.4743	1.8149
M7	0.6595	1.4848	1.8296
M8	0.6654	1.4738	1.8136

The majority of stand-level predictor variables were significantly correlated with the coefficient β_1 in M4 (Equation (19)), while no stand-level predictor variable was significantly correlated with the parameter β_2 of Equation (19) (Figure 2). Finally, only the DH was retained in the generalized H-D model (Equation (20)) according to the two-step linear regression and correlation (Figure 3). The generalized model obtained significantly better goodness-of-fit ($p < 0.0001$):

$$H_{ij} = 1.3 + \beta_1 \exp(\beta_2/D_{ij}) + \varepsilon_{ij} \tag{19}$$

$$H_{ij} = 1.3 + (\beta_1 + \beta_2 DH_i) \exp(\beta_3/D_{ij}) + \varepsilon_{ij} \tag{20}$$

where H_{ij} and D_{ij} are the H and D of tree j in plot i, respectively; β_1, β_2, β_3 are fixed coefficients; DH_i is the dominant height for sample plot i; and ε_{ij} is the error.

3.2. Mixed-Effects Models

The final formulas for the basic and generalized mixed-effects models are presented in Equations (21) and (22), respectively. The power variance function showed better performance than the other two functions in removing the heteroscedasticity of basic and generalized mixed-effects models (Table S1, Figure 4).

$$H_{ij} = 1.3 + (\beta_1 + u_i) \exp(\beta_2/D_{ij}) + \varepsilon_{ij} \tag{21}$$

$$H_{ij} = 1.3 + (\beta_1 + u_i + \beta_2 DH_i] \exp(\beta_3/D_{ij}) + \varepsilon_{ij} \tag{22}$$
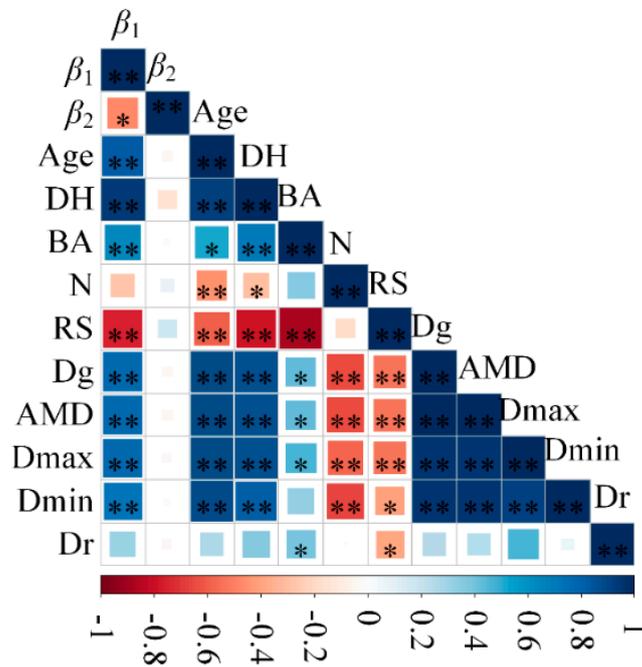
Figure 2. Correlation among the stand-level predictor variables and basic height-diameter (H-D) model coefficients. Age is the stand age; DH is the dominant height; BA is the basal area per hectare; N is the stand density; RS is the relative spacing; Dg is the stand quadratic mean diameter; AMD is the stand arithmetic mean diameter; Dmax and Dmin are the maximum and minimum diameter of the plot, respectively; Dr is diameter range per plot; β_1 and β_2 are model parameters. * and ** denote the significant correlation at level of $p < 0.05$ and $p < 0.01$, respectively.
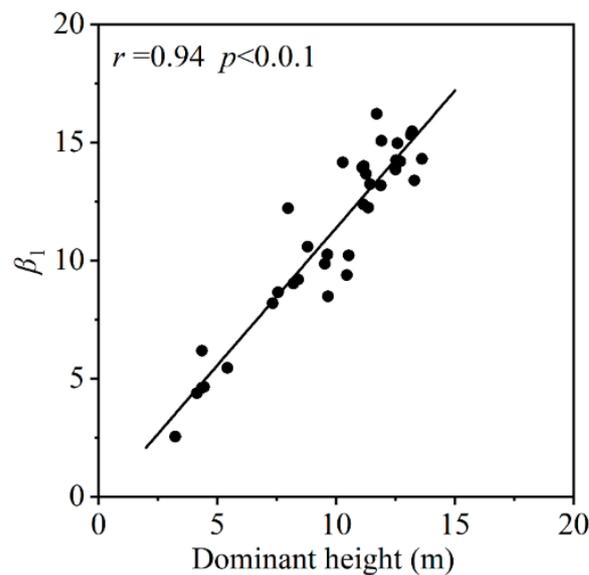
Figure 3. Linear relationship between the dominant height (DH) and the estimated coefficient β_1 of the basic model. R means the Pearson correlation between the model coefficient β_1 of the basic model and dominant height.
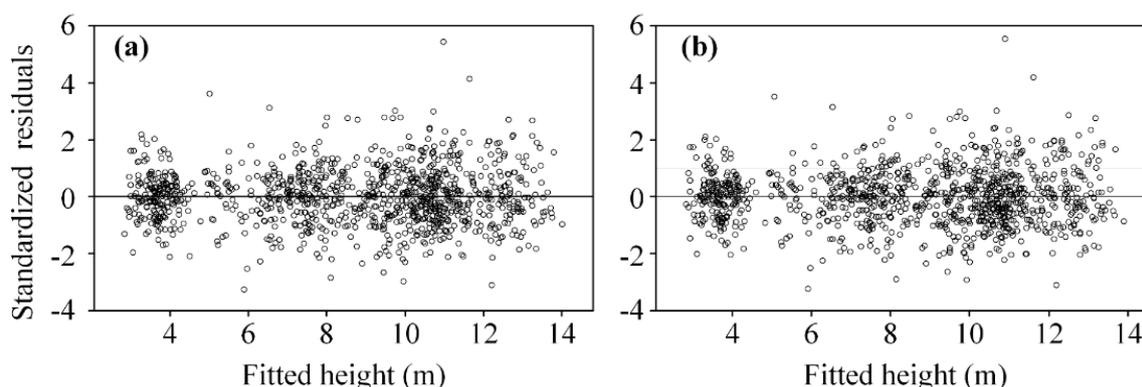
Figure 4. Distributions of standardized residuals of basic (**a**,**b**) generalized mixed-effects models.
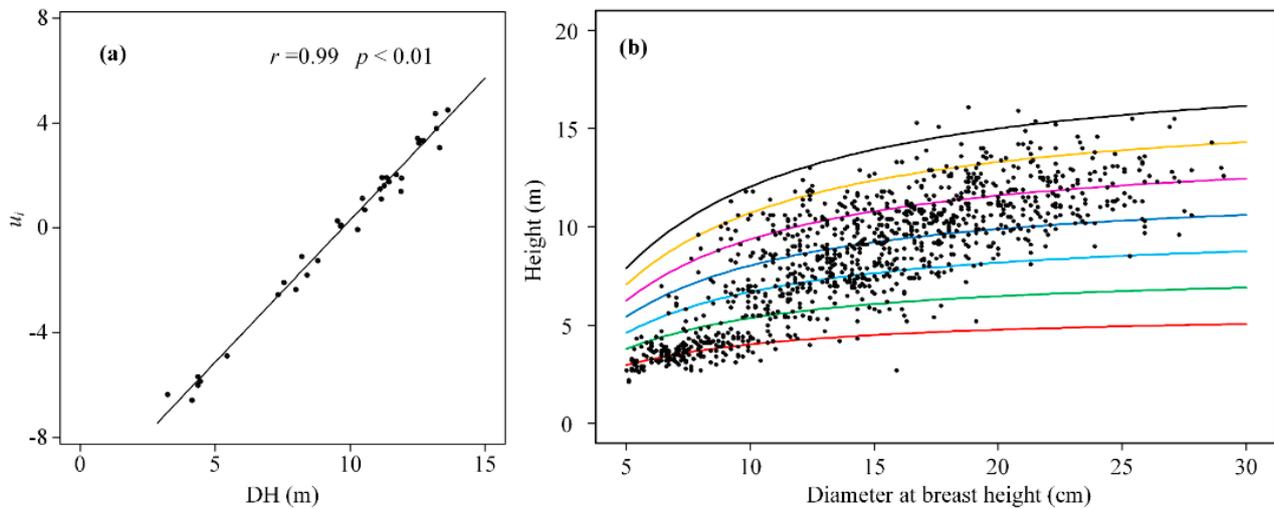
The generalized mixed-effects model had lower AIC and BIC values than the basic mixed-effects model (Table 4) and was also significantly different from the basic mixed-effects model ($p < 0.0001$). The random-effects values of the basic mixed-effects model were larger than those of generalized mixed-effects model, as the standard deviation of the basic model random-effects was ten times larger than that of the generalized mixed-effects model. The DH explains the high variety of H-D relationships as random-effects do (Figure 5a). In addition, for the trees with same diameter, the tree height increased with the increase in the stand DH (Figure 5b). The basic mixed-effects model, however, had an almost similar (even better) performance as the generalized mixed-effects model according to the R^2, MAE, and RMSE (Table 4). In addition, the two mixed-effects model had similar distributions of the standardized residuals (Figure 4).

Table 4. Parameter estimations and statistical criteria of the basic and generalized mixed-effects models.

Parameters	Basic	Generalized
β_1	10.76302	0.06223
β_2	−5.06705	1.08832
β_3		−4.86799
σ_u	3.25644	0.30297
δ	0.50097	0.48913
σ	0.21609	0.22302
AIC	2746.0	2610.8
BIC	2770.9	2640.6
R^2	0.933	0.932
MAE	0.612	0.615
RMSE	0.810	0.815

3.3. Mixed-Effects H-D Model Calibration

The random-effects predictions had lower MAE and RMSE values than the fixed predictions and were closer to the observed height values (Figures 6a,b and 7a,b, Table S2). The generalized mixed-effects model had better prediction accuracy than the basic mixed-effects model under the majority of calibration designs with lower MAE and RMSE (Figure 6a,b, Table S2). In addition, the number and the diameter size of measured trees had an obvious effect on the prediction accuracy of the basic mixed-effects model; however, this was not the case for the generalized H-D model (Table S2). The calibration accuracy increased with the increase in prior height measurements; nevertheless, the accuracy increased negligibly (<5 cm) when the number of trees used for calibration was higher than two (Table S2).

Figure 5. The Pearson correlation between DH and the estimated random-effects of the basic mixed-effects model (**a**), and effects of DH on H-D relationships (**b**). The fitted H-D curves in the figure (**b**) were based on the fixed prediction of the generalized mixed-effects model. The colorful lines from the bottom to the top indicate DH values from 4 m (**red**) to 16 m (**black**) with a step of 2 m. The scatters in (**b**) are the H-D measurements.
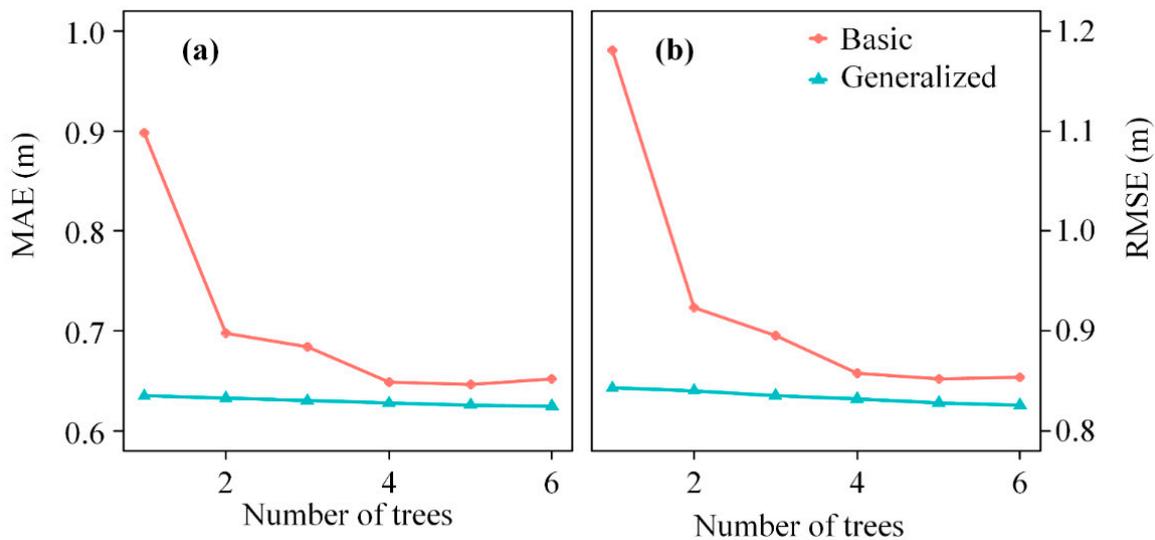
Figure 6. Changes in the mean absolute error (MAE; (**a**)) and the root mean squared error (RMSE; (**b**)) values obtained with an increasing number of subsample trees. Only the best results are shown for equal tree number.

Although the best design with different tree number showed that increasing height measurements improved the prediction accuracy of the basic mixed-effects model, the improvement of accuracy was negligible (<5 cm) when the number of measured trees was more than two (Table S2). In addition, the difference in random-effects prediction performances between the two mixed-effects models was very small when more than two tree heights were available (Figure 6a,b).

To obtain an increased prediction accuracy with as few prior tree height measurements as possible, the basic mixed-effects model required the two thickest trees' heights, and the generalized mixed-effects model required one medium tree height. The two mixed-effects models had similar predicted results based on the above-mentioned designs (Figure 7).
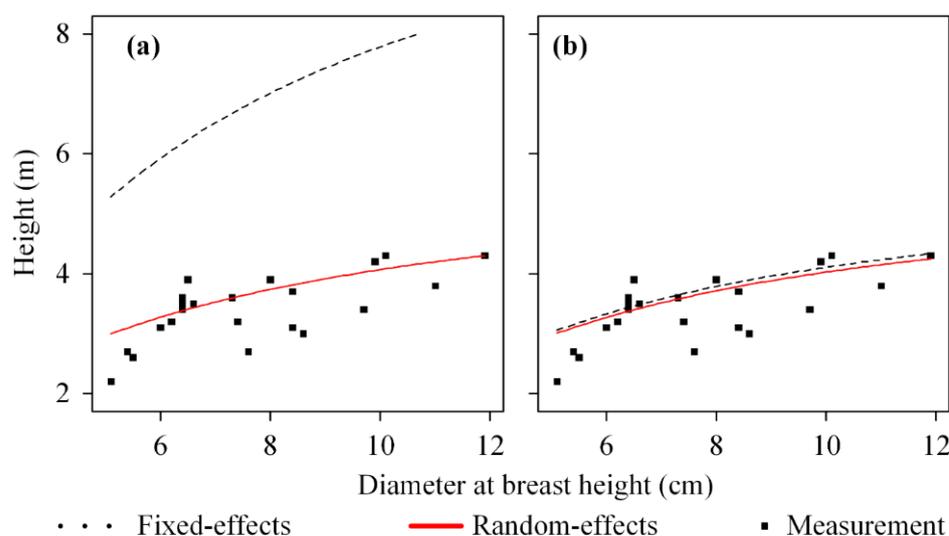
Figure 7. Fixed- and random-effects prediction performance for the basic (**a**) and generalized (**b**) mixed-effects models in an example plot. (**a**) The curve of random-effects prediction of basic mixed-effects model was based on calibration with the two thickest trees. (**b**) The curve of random-effects prediction was based on calibration with one medium-sized tree.

4. Discussion

4.1. Basic and Generalized H-D Models

Our study's aim was to build an appropriate H-D model for the Mongolian pine stands in Zhanggutai, where the oldest introduced sandy Mongolian pine plantations in China are located [44]. We developed a mixed-effects H-D model and determined an economical method and a simple but effective calibration design for estimating the heights of Mongolian pines to aid forest management planning [6,21].

We used equations with only two coefficients due to the nonconvergence associated with three-parameter models when fitting each plot. Nonconvergence issues have been reported in several studies that aimed to develop nonlinear H-D models [6,27,45]. Among the eight candidate models, the model of Schumacher [35] was selected as the basic H-D model because it had the highest accuracy compared to the other seven models tested. This model has also chosen as a basic mathematical form, suitable for further improvement in similar research studies [7,21,27].

Developing generalized H-D models is an effective solution to account for the variations in H-D relationships among plots. Generalized H-D models can broaden the applications of such predictive tools. In this work, we used a common method to link the parameters and stand-level predictor variables and to develop the generalized H-D model [38]. This method has been widely employed to build generalized H-D models [6,7,15]. Among the tested stand-level predictor variables, only the DH was determined to be necessary for developing the generalized H-D model. DH positively and significantly correlated with the asymptote coefficient of the candidate model. Our results are consistent with those of many previous studies [5,17,21,46] that used dominant height as an additional stand-level predictor variable associated with the asymptote coefficient. This could be expected, as DH is a measure of the stand's maximum height potential, which is usually associated with the site productivity and the site index [47]. DH is a result of the competition process present in the stand, the stand density [17], and the stand quality [5,46]. Although significant correlation between the coefficients and the stand-level predictor variables such as the Age, the N and the BA was detected, adding other stand-level predictor variables did not increase the model performances significantly. In this work, DH was significantly correlated with other stand-level predictor variables except the Dr. One of the reasons why the stand-level predictor variables that were significantly correlated with the coefficient of the basic model did not significantly improve the H-D model is the multicollinearity. Finally,

the generalized H-D model containing only DH as an additional stand-level predictor variable was chosen as the best generalized H-D model for the Mongolian pine stands in the study area.

4.2. Mixed-Effects H-D Models

Mixed-effects models are more flexible and have a better predictive accuracy in modeling H-D relationships. Furthermore, they can considerably improve the model prediction accuracy through different calibration scenarios [17,24]. In this study, the basic mixed-effect model includes only one random-effect parameter. In addition, the random-effect was included in the asymptote coefficient, which indicated that the maximum height had a high variation between stands [17,25].

Generalized mixed-effects models containing stand-level predictor variables that can account for the variability among stands generally achieve a better performance in fitting and prediction [27]. The DH stand-level predictor variable significantly improved the basic model according to the AIC and the BIC statistics. In addition, the generalized mixed-effects model also had better predictive accuracy under the majority of calibration designs, which is consistent with our first hypothesis. Nevertheless, the MAE and RMSE statistics did not improve based on the results of fitting and the best calibration design. Similar results were described by Zang et al. (2016), where their basic mixed-effects H-D model for larch plantations in northern and northeastern China provided better RMSE and MAE statistics than the generalized mixed-effects model [25]. Furthermore, Huang et al. (2009) found that the basic mixed-effects model performed as well as or better than the generalized mixed-effects model for aspen grown in boreal mixed wood forests in Alberta, which is similar to our study in that the inclusion of other stand-level predictor variables in the model did not produce more accurate predictions [5]. A possible explanation for why the generalized mixed-effects H-D model does not provide a better accuracy compared to the basic mixed-effects model is related to the capacity of the random-effect parameter to explain the various H-D relationships among stands. Our results support this theory, since both the stand-level predictor variable and the random effects were introduced alongside the asymptote coefficient of the model, and the random effects of the basic mixed-effects model were significantly larger than those of the generalized mixed-effects model. In addition, the estimated random-effects of the basic mixed-effects model were strongly and significantly correlated with the stand-level predictor variable DH. However, generalized models provide a better biological representation of the process studied and are helpful for large datasets [25,48]. Thus, whether or not the generalized mixed-effects model is still needed to predict the height for the sandy Mongolian pine plantations needs further investigation. Furthermore, DH models are not yet available for this species as far as we know. This would make the generalized model even more difficult to apply as the field work would involve measuring the height of the tallest 100 trees or estimating DH based on a sample plot. Considering that similar results can be obtained with the basic mixed-effects model by predicting the random effect with a reduced number of trees measured on the field, we recommend the use of the basic mixed-effects model to reduce the necessary field work.

4.3. Calibration for Random-Effects Prediction

The most important reason for developing H-D models is to estimate tree heights using tree diameters at breast height for new stands where these measurements are missing. The mixed-effects framework allows for the calibration of the fixed effects in order to obtain more accurate predictions than the ones obtained using the population mean coefficients. The calibration design (which mainly includes the number and the size of trees) has an obvious effect on the predictive accuracy of the model [49–51]. Furthermore, the appropriate calibration design varies between forest types and species [6,7,17]. To determine the most appropriate prediction strategy, 47 calibration combinations were analyzed separately for the generalized and basic mixed-effects H-D models. As we sug-

gested in our second hypothesis, the MAE and the RMSE decreased with the increase in the number of trees. However, the decrease in the MAE and the RMSE was different for each of the two mixed-effects models. The MAE and the RMSE for the generalized mixed-effects H-D model fluctuated slightly when changing the sampling design, similar to Crecente-Campo et al. (2014) [52]. Unlike the generalized mixed-effects H-D model, the basic mixed-effects H-D model requires less field work effort and costs for random-effects calibration. Moreover, the basic mixed-effects H-D model does not require the extra DH measurement in addition to the D measurements. Moreover, the basic mixed-effects H-D model required only two height measurements for the calculation of random parameters and yielded similar MAE and RMSE when compared with the generalized mixed-effects model.

Unexpectedly, the third hypothesis was not confirmed for our dataset: sampling trees from different diameter classes did not lead to better calibration results. In this study, we found that for the basic mixed-effects H-D model only the two thickest tree heights are needed in order to calibrate the model and to produce reliable and labor-saving predictions. In contrast to our findings, Bronisz and Mehtätalo (2020) reported that using the extreme trees (the thinnest and the thickest) in *Betula pendula* stands led to better performances than any other analyzed sampling strategy [7]. Our results are close to those obtained by Ciceu et al. (2020), who reported that the diameter at breast height of medium and thickest trees was the best for model calibration for *Picea abies* in mixed uneven-aged stands [6].

Our basic mixed-effects model calibration results can be explained based on the strong correlation between the DH and the asymptote coefficient. The heights of the two largest trees serve as an estimation of the dominant height, thus replacing the importance of this variable in the H-D relationship between stands. Thus, using the thickest trees provided more additional information for calibrating the basic mixed-effects model than the information brought by the median tree height in the generalized H-D model.

Following the above-mentioned results, we recommend sampling the two thickest trees per plot for a practical height prediction. However, our study was only conducted in Mongolian pine plantations on sandy land in northwest Liaoning Province of China, as there was no available independent dataset for model validation in other regions. These results should be carefully validated when the region is changed.

5. Conclusions

Basic and generalized H-D models were obtained for the Mongolian pine plantations on sandy land in northern China. We found that the Schumacher [37] model had the best performance in modeling local H-D relationships relative to the other basic H-D models found in the literature. The DH was found to significantly describe the H-D relationships among plots.

We found that the basic mixed-effects H-D model produces similar prediction to the generalized model but with less sampling effort, which is more suitable for practical applications. Thus, the basic mixed-effects H-D model can be a solution for easily estimating individual tree heights in Mongolian pine plantations where DH measurements are not available. Furthermore, an increased accuracy could be obtained by calibrating the random effects using additional height measurements of the two thickest trees.

The use of this method would reduce the necessary field work which would otherwise be needed to determine the dominant height of the stand by measuring only two heights for random effects calibration.

References

1. Zagas, T.D.; Raptis, D.I.; Zagas, D.T. Identifying and mapping the protective forests of southeast Mt. Olympus as a tool for sustainable ecological and silvicultural planning, in a multi-purpose forest management framework. *Ecol. Eng.* **2011**, *37*, 286–293. [CrossRef]
2. Zhu, J.-J. A review of the present situation and future prospect of science of protective forest. *Chin. J. Plant Ecol.* **2013**, *37*, 872–888. [CrossRef]
3. Zhu, J.; Zheng, X. The prospects of development of the Three-North Afforestation Program (TNAP): On the basis of the results of the 40-year construction general assessment of the TNAP. *Chin. J. Ecol.* **2019**, *38*, 1600–1610. [CrossRef]
4. Song, L.; Li, M.; Zhu, J.; Zhang, J. Comparisons of radial growth and tree-ring cellulose δ^{13}C for *Pinus sylvestris* var. *mongolica* in natural and plantation forests on sandy lands. *J. For. Res.* **2017**, *22*, 160–168. [CrossRef]
5. Huang, S.; Wiens, D.P.; Yang, Y.; Meng, S.X.; Vanderschaaf, C.L. Assessing the impacts of species composition, top height and density on individual tree height prediction of quaking aspen in boreal mixed woods. *For. Ecol. Manag.* **2009**, *258*, 1235–1247. [CrossRef]
6. Ciceu, A.; Garcia-Duro, J.; Seceleanu, I.; Badea, O. A generalized nonlinear mixed-effects height-diameter model for Norway spruce in mixed-uneven aged stands. *For. Ecol. Manag.* **2020**, *477*, 118507. [CrossRef]
7. Bronisz, K.; Mehtätalo, L. Mixed-effects generalized height–diameter model for young silver birch stands on post-agricultural lands. *For. Ecol. Manag.* **2020**, *460*, 117901. [CrossRef]
8. Curtis, R.O. Height-diameter and height-diameter-age equations for second-growth Douglas-fir. *For. Sci.* **1967**, *13*, 365–375.
9. Moore, J.A.; Zhang, L.; Dean, S. Height-diameter equations for ten tree species in the Inland Northwest. *West. J. Appl. For.* **1996**, *11*, 132–137. [CrossRef]
10. Wang, X.; Fang, J.; Tang, Z.; Zhu, B. Climatic control of primary forest structure and DBH-height allometry in Northeast China. *For. Ecol. Manag.* **2006**, *234*, 264–274. [CrossRef]
11. Aishan, T.; Halik, U.; Betz, F.; Gaertner, P.; Cyffka, B. Modeling height-diameter relationship for *Populus euphratica* in the Tarim riparian forest ecosystem, Northwest China. *J. For. Res.* **2016**, *27*, 889–900. [CrossRef]
12. Huang, S.; Titus, S.J.; Wiens, D.P. Comparison of nonlinear height-diameter functions for major Alberta tree species. *Can. J. For. Res.* **1992**, *22*, 1297–1304. [CrossRef]
13. Zhang, X.; Chhin, S.; Fu, L.; Lu, L.; Duan, A.; Zhang, J. Climate-sensitive tree height-diameter allometry for Chinese fir in southern China. *Forestry* **2019**, *92*, 167–176. [CrossRef]
14. Schmidt, M.; Breidenbach, J.; Astrup, R. Longitudinal height-diameter curves for Norway spruce, Scots pine and silver birch in Norway based on shape constraint additive regression models. *For. Ecosyst.* **2018**, *5*, 9. [CrossRef]
15. Sharma, M. Height-diameter relationships of boreal tree species grown in plantations and natural stands. *For. Sci.* **2016**, *62*, 70–77. [CrossRef]
16. Lappi, J. A longitudinal analysis of height/diameter curves. *For. Sci.* **1997**, *43*, 555–570.
17. Raptis, D.I.; Kazana, V.; Kazaklis, A.; Stamatiou, C. Mixed-effects height–diameter models for black pine (*Pinus nigra* Arn.) forest management. *Trees* **2021**, *35*, 1167–1183. [CrossRef]
18. Calama, R.; Montero, G. Interregional nonlinear height-diameter model with random coefficients for stone pine in Spain. *Can. J. For. Res.* **2004**, *34*, 150–163. [CrossRef]
19. Sharma, M.; Zhang, S.Y. Height-diameter models using stand characteristics for *Pinus banksiana* and *Picea mariana*. *Scand. J. For. Res.* **2004**, *19*, 442–451. [CrossRef]
20. Sacramento, C.-R.; Gabriel, A.-G.J.; Felipe, C.-C.; Javier, C.-R.J. Local and generalized height-diameter models with random parameters for mixed, uneven-aged forests in Northwestern Durango, Mexico. *For. Ecosyst.* **2014**, *1*, 41–49.
21. Gomez-Garcia, E.; Fonseca, T.F.; Crecente-Campo, F.; Almeida, L.R.; Dieguez-Aranda, U.; Huang, S.; Marques, C.P. Height-diameter models for maritime pine in Portugal: A comparison of basic, generalized and mixed-effects models. *Iforest Biogeosci. For.* **2016**, *9*, 72–78. [CrossRef]

22. Lebedev, A.V. New generalised height-diameter models for the birch stands in European Russia. *Balt. For.* **2020**, *26*, 265–271. [CrossRef]

23. Temesgen, H.; Gadow, K.V. Generalized height-diameter models—An application for major tree species in complex stands of interior British Columbia. *Eur. J. For. Res.* **2004**, *123*, 45–51. [CrossRef]

24. Fang, Z.X.; Bailey, R.L. Nonlinear mixed effects modeling for slash pine dominant height growth following intensive silvicultural treatments. *For. Sci.* **2001**, *47*, 287–300.

25. Zang, H.; Lei, X.; Zeng, W. Height-diameter equations for larch plantations in northern and northeastern China: A comparison of the mixed-effects, quantile regression and generalized additive models. *Forestry* **2016**, *89*, 434–445. [CrossRef]

26. Sharma, R.P.; Vacek, Z.; Vacek, S.; Kučera, M. Modelling individual tree height–diameter relationships for multi-layered and multi-species forests in central Europe. *Trees* **2019**, *33*, 103–119. [CrossRef]

27. Mehtätalo, L.; De-Miguel, S.; Gregoire, T.G. Modeling height-diameter curves for prediction. *Can. J. For. Res.* **2015**, *45*, 826–837. [CrossRef]

28. Pinheiro, J.C.; Bates, D.M. *Mixed-Effects Models in S and S-PLUS*; Springer: New York, NY, USA, 2000; pp. 273–301.

29. Zhou, Y.; Lei, Z.; Zhou, F.; Han, Y.; Yu, D.; Zhang, Y. Impact of climate factors on height growth of *Pinus sylvestris* var. *mongolica*. *PLoS ONE* **2019**, *14*, e0213509. [CrossRef] [PubMed]

30. Lei, Z.; Yu, D.; Zhou, F.; Zhang, Y.; Yu, D.; Zhou, Y.; Han, Y. Changes in soil organic carbon and its influencing factors in the growth of *Pinus sylvestris* var. *mongolica* plantation in Horqin Sandy Land, Northeast China. *Sci. Rep.* **2019**, *9*, 16453. [CrossRef] [PubMed]

31. Lei, X.; Peng, C.; Wang, H.; Zhou, X. Individual height-diameter models for young black spruce (*Picea mariana*) and jack pine (*Pinus banksiana*) plantations in New Brunswick, Canada. *Forest. Chron.* **2009**, *85*, 43–56. [CrossRef]

32. Stoffels, A.; Soest, J.V. The main problems in sample plots. *Ned. Bosbouwtijdschr.* **1953**, *25*, 190–199.

33. Schumacher, F.X. A new growth curve and its application to timber yield studies. *J. For.* **1939**, *37*, 817–820.

34. Wykoff, W.R.; Crookston, N.L.; Stage, A.R. *User's Guide to the Stand Prognosis Model*; United States Department of Agriculture: Washington, DC, USA, 1982; Volume 133. [CrossRef]

35. Bates, D.M.; Watts, D.G. Relative curvature measure of nonlinearity. *J. R. Stat. Soc. Ser. B* **1980**, *42*, 1–25. [CrossRef]

36. Larson, B.C. Development and growth of even-aged stands of Douglas-fir and grand fir. *Can. J. For. Res.* **1986**, *16*, 367–372. [CrossRef]

37. Meyer, H.A. A Mathematical Expression for Height Curves. *J. For. Res.* **1940**, *38*, 415–420.

38. Staudhammer, C.; Lemay, V. Height prediction equations using diameter and stand density measures. *For. Chron.* **2000**, *76*, 303–309. [CrossRef]

39. Trincado, G.; Vanderschaaf, C.L.; Burkhart, H.E. Regional mixed-effects height-diameter models for loblolly pine (*Pinus taeda* L.) plantations. *Eur. J. For. Res.* **2007**, *126*, 253–262. [CrossRef]

40. R Development Core Team. R: A Language and environment for statistical computing. In *R Foundation for Statistical Computing*; R Development Core Team: Vienna, Austria, 2020.

41. Pinheiro, J.; Bate, D.; Debroy, S.; Sarkar, D.; Team, R.C. nlme: Linear and Nonlinear Mixed Effects Models. In *R Package Version*; Available online: https://CRAN.R-project.org/package=nlme (accessed on 4 February 2021).

42. Vonesh, E.F.; Chinchilli, V.M. *Linear and Nonlinear Models for the Analysis of Repeated Measurements*; CRC Press: Boca Raton, FL, USA, 1997. [CrossRef]

43. Nigh, G. Calculating empirical best linear unbiased predictors (EBLUPs) for nonlinear mixed effects models in Excel/Solver. *Forest. Chron.* **2012**, *88*, 340–344. [CrossRef]

44. Song, L.; Zhu, J.; Li, M.; Zhang, J. Water use patterns of *Pinus sylvestris* var. *mongolica* trees of different ages in a semiarid sandy lands of Northeast China. *Environ. Exp. Bot.* **2016**, *129*, 94–107. [CrossRef]

45. Yang, Y.; Huang, S. On the statistical and biological behaviors of nonlinear mixed forest models. *Eur. J. For. Res.* **2013**, *132*, 727–736. [CrossRef]

46. Sharma, R.P.; Breidenbach, J. Modeling height-diameter relationships for Norway spruce, Scots pine, and downy birch using Norwegian national forest inventory data. *For. Sci. Technol.* **2015**, *11*, 44–53. [CrossRef]

47. Avery, T.E.; Burkhart, H. *Forest Measurements*; Waveland Press Inc.: Long Grove, IL, USA, 2015; pp. 312–352.

48. Yang, Y.; Huang, S. Allometric modelling of crown width for white spruce by fixed-and mixed-effects models. *For. Chron.* **2017**, *93*, 138–147. [CrossRef]

49. Temesgen, H.; Monleon, V.J.; Hann, D.W. Analysis and comparison of nonlinear tree height prediction strategies for Douglas-fir forests. *Can. J. For. Res.* **2008**, *38*, 553–565. [CrossRef]

50. Fu, L.; Zhang, H.; Sharma, R.P.; Pang, L.; Wang, G. A generalized nonlinear mixed-effects height to crown base model for Mongolian oak in northeast China. *For. Ecol. Manag.* **2017**, *384*, 34–43. [CrossRef]

51. Fu, L.Y.; Sharma, R.P.; Hao, K.J.; Tang, S.Z. A generalized interregional nonlinear mixed-effects crown width model for Prince Rupprecht larch in northern China. *For. Ecol. Manag.* **2017**, *389*, 364–373. [CrossRef]

52. Crecente-Campo, F.; Corral-Rivas, J.J.; Vargas-Larreta, B.; Wehenkel, C. Can random components explain differences in the height-diameter relationship in mixed uneven-aged stands? *Ann. For. Sci.* **2014**, *71*, 51–70. [CrossRef]